Design of LDPC Codes for Slepian-Wolf coding with uncertain knowledge of the correlation

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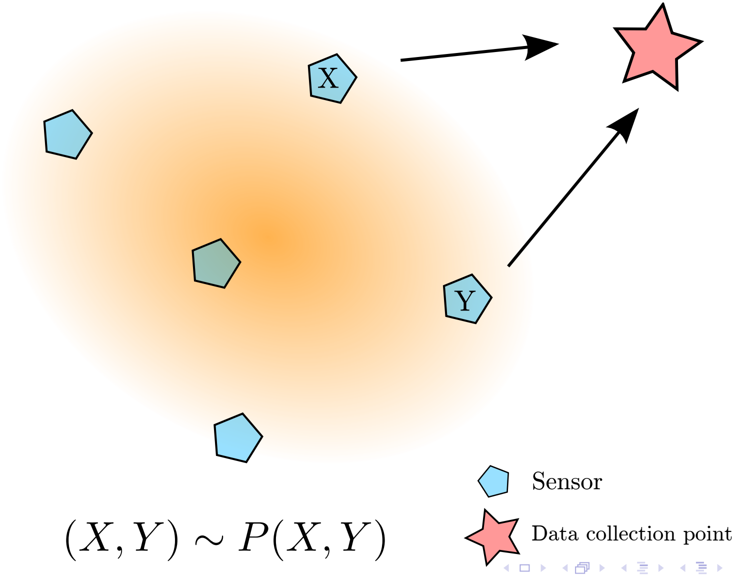
Context



$$(X, Y) \sim P(X, Y)$$

Context

- Theoretical performance [SW73]
- Coding schemes based on channel codes [LXG02, LXG03, MUM10, XLC04]

In general, perfect knowledge of $P(X, Y)$

- $P(X)$ unknown [JVW10]
- $P(Y|X)$ given at decoder, partly unknown at encoder [Sga77]

Practical solution

- Feedback channel [AZG02, EY05, VAG06]

Context

In this work

- $P(Y|X)$ partly unknown to both encoder and decoder
- No feedback

Objectives

- Design efficient coding/decoding schemes robust to uncertainty on $P(Y|X)$
- Solution based on non-binary LDPC codes

Modeling the uncertainty

Four source models considered in [DRK12].
Here, focus on the Static without Prior Source (SwP-Source)

Definition (SwP-Source)

A SwP-Source (X, Y), *produces a sequence of independent discrete symbols* $\{(X_n, Y_n)\}_{n=1}^{+\infty}$ *drawn from a distribution belonging to*

$$\{P(X, Y|\theta) = P(X)P(Y|X, \theta)\}_{\theta \in \mathscr{P}_\theta}$$

θ *is fixed for* $\{(X_n, Y_n)\}_{n=1}^{+\infty}$.

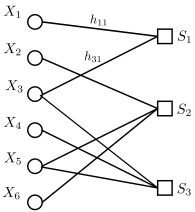
LDPC-based Encoder in GF(q)

Theoretical Performance [Csi82]

$$R = \sup_{\theta \in \mathscr{P}_\theta} H(X|Y,\theta).$$

Encoding in GF(q) ($m < n$)

$$\mathbf{s}^m = H^T \mathbf{x}^n$$



Degree distributions $(\lambda(x), \rho(x))$ for H dimensioned for the worst case, need to be optimized

Decoder

Two-step EM-based approach: $(\mathbf{x}^{(\ell)}, \theta^{(\ell)})$ at iteration ℓ

- LDPC decoding with estimate $\theta^{(\ell)}$ to obtain

$$P(X_n = k | y_n, \mathbf{s}, \theta^{(\ell)})$$

- Update of the $\theta^{(\ell)}$ by maximizing

$$Q(\theta, \theta^{(\ell)}) = E_{\mathbf{X}|\mathbf{y},\mathbf{s},\theta^{(\ell)}} [\log P(\mathbf{y}|\mathbf{X}, \mathbf{s}, \theta)]$$

$$= \sum_{n=1}^{N} \sum_{k=0}^{q-1} P(X_n = k | y_n, \mathbf{s}, \theta^{(\ell)}) \log P(y_n | X_n = k, \theta)$$

Decoder

Two-step EM-based approach

- LDPC decoding with estimate $\theta^{(\ell)}$ to obtain

$$P(X_n = k | y_n, \mathbf{s}, \theta^{(\ell)})$$

- Update of the $\theta^{(\ell)}$ for $Y = X \oplus Z$, $P(Z = k) = \theta_k$

$$\theta_k^{(\ell+1)} = \frac{\sum_{n=1}^{N} P(X_n = y_n \ominus k | y_n, \mathbf{s}, \theta^{(\ell)})}{\sum_{n=1}^{N} \sum_{k'=0}^{q-1} P(X_n = y_n \ominus k' | y_n, \mathbf{s}, \theta^{(\ell)})}$$

Initialization of the EM algorithm (Additive Model)

Additive model: $Y = X \oplus Z$, $P(Z = k) = \theta_k$

- Compute

$$\mathbf{u} = \mathbf{s} \ominus H^T \mathbf{y} = H^T \mathbf{z}$$

Assumption: the U_m are obtained from *i.i.d.* R.V.s $Z_j^{(m)}$.

- Maximize

$$L(\theta) = \log P(\mathbf{u}|\theta) = \sum_{m=1}^{M} \log \mathscr{F}_{u_m}^{-1} \left(\prod_{j=1}^{dc} \mathscr{F}(W[h_j^{(m)}]\theta) \right)$$

Experimental framework

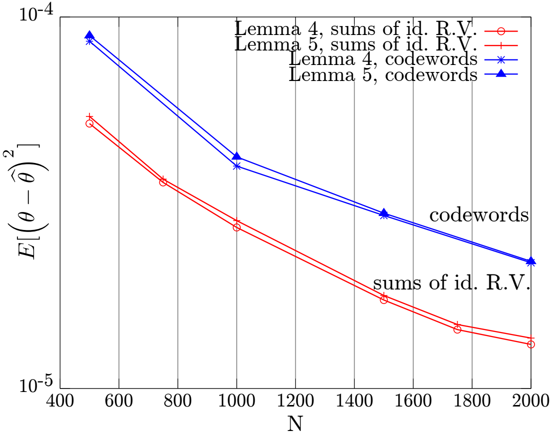
Symbols in GF(4), $Y = X \oplus Z$,
$\theta = [\theta_0, \dots, \theta_3]$ and $Pr(Z = k) = \theta_k$.
$\mathscr{P}_\theta$ s.t. $\forall \theta \in \mathscr{P}_\theta,\ \theta_0 \geq 0.76$.

Code tuned for the worst case $\theta = [0.76, 0.08, 0.08, 0.08]$

- $\lambda(x) = 0.413x + 0.375x^2 + 0.012x^4$
- $\rho(x) = x$
- $R = 1.6$ bit/symbol

Initialization of the EM algorithm

MSE of the estimators



Global scheme

1000 source vectors of length 1000 are generated.
For each vector, θ selected uniformly at random in $\mathscr{P}_\theta$.

Setup	Err	Time (s)	Rate (bit/symb.)
Genie-aided	$< 10^{-5}$	5.4	1.6
Learn. Seq.	$< 10^{-5}$	4.2	1.7
EM	$< 10^{-5}$	9.1	1.6
EM random	7.2×10^{-3}	47.0	1.6

Conclusions

Summary

- Practical coding scheme based on non-binary LDPC codes when the correlation is uncertain

Future works

- Extension to the lossy case
- Design of good degree distributions
- Correlation model selection

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