Source Coding with Side Information at the Decoder: Models with Uncertainty, Performance Bounds, and Practical Coding Schemes

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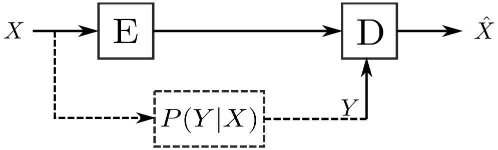
Introduction



- If $P(X, Y)$ perfectly known
  - Performance : $R^{SW}_{X|Y} = H(X|Y)$ bits/symbol
  - Practical Solution : LDPC codes
- If $P(X, Y)$ not perfectly known
  - Performance : ?
  - Practical Solution : ?

Outline

Time Invariant Parameters

Unknown parameter $\theta \in \mathcal{P}_\theta$ fixed for a sequence $\{X_n, Y_n\}_{n=1}^{+\infty}$, varying from sequence to sequence

Models

- P-Source : $(X, Y) \sim P(X, Y|\theta)$, prior $\Theta \sim P_\Theta(\theta)$
- WP-Source : $(X, Y) \sim P_\theta(X, Y)$

Stationary, non-ergodic models

Example : X BSS, $Y|X$ BSC(θ)

Time Varying Parameters

Unknown sequence of parameters $\{\pi_n\}_{n=1}^{+\infty}$, $\pi_n \in \mathcal{P}_\pi$

Models

- M-Source : $(X_n, Y_n) \sim P(X_n, Y_n|\pi_n)$, prior $\Pi \sim P_\Pi(\pi)$
- WPM-Source : $(X_n, Y_n) \sim P_{\pi_n}(X_n, Y_n)$

M-Source stationary and ergodic
WPM-Source non-stationary and non-ergodic

Example : X_n BSS, $Y_n|X_n$ BSC(π_n)

Performance

- P-Source: Information Spectrum approach [Han03]

$$R^{P}_{X|Y} = P_{\Theta}\text{-ess.}\sup H(X|Y, \Theta = \theta)$$

- WP-Source: Universal Slepian-Wolf [Csi82]

$$R^{WP}_{X|Y} = \sup_{\theta \in \mathcal{P}_\theta} H_\theta(X|Y)$$

Practical coding scheme (P-Source)

X : BSS, $Y|X$: BSC(θ), sequence $\mathbf{X}_1^N$ to encode as $(\mathbf{X}_1^{u_N}, \mathbf{Z}_1^M)$

- First step :

Learning sequence $\mathbf{X}_1^{u_N}$

- Second step :

LDPC coding of $\mathbf{X}_{u_N+1}^N$ as $\mathbf{Z}_1^M = H^T \mathbf{X}_{u_N+1}^N$

where H : degree distributions $(\lambda(x), \rho(x))$

Question

How to choose the length u_N of the learning sequence?

Practical coding scheme (P-Source)

The LDPC decoder requires $\theta = P(X = 1|Y = 0)$

- Decoding : LDPC decoder initialized with

$$\theta_r := P(X_n = 1|Y_n = 0, \mathbf{x}_1^{u_N}, \mathbf{y}_1^{u_N}) = \int_{\theta \in \mathcal{P}_\theta} P(X_n|Y_n, \theta) P_{\Theta|\mathbf{x}_1^{u_N}, \mathbf{y}_1^{u_N}}(\theta) d\theta$$

mismatch on the parameter $(\theta_r \neq \theta)$

- Performance : Density Evolution initialized with

$$p_X(x) = \theta\delta\left(x + \log\frac{1-\theta_r}{\theta_r}\right) + (1-\theta)\delta\left(x - \log\frac{1-\theta_r}{\theta_r}\right)$$

Practical coding scheme (P-Source)

γ : outage measure

1. Outage set $B^{\gamma}_{\theta|\theta_r}$ s.t. $\int_{\theta \in \mathcal{B}^{\gamma}_{\theta|\theta_r}} P_{\Theta|\theta_r}(\theta) d\theta > 1 - \gamma$
2. Error Probability : $P_e(\lambda, \rho, \theta, \theta_r)$
3. Set of admissible parameters (λ, ρ) :

$$\Gamma(\gamma, \epsilon_t) = \left\{ (\lambda, \rho) \text{ s.t. } \forall \theta_r \in \text{Conv}(\mathcal{P}_\theta), \exists B^{\gamma}_{\theta|\theta_r} : \forall \theta \in B^{\gamma}_{\theta|\theta_r}, P_e(\lambda, \rho, \theta, \theta_r) < \epsilon_t \right\}$$

Total rate

$$R^N(\gamma, \varepsilon_t) = \frac{u_N}{N} H(X) + \frac{N - u_N}{N} \inf_{(\lambda,\rho) \in \Gamma(\gamma, \epsilon_t)} R(\lambda, \rho)$$

Performance

- M-Source: Classical Slepian-Wolf [SW73]

$$R^{M}_{X|Y} = H(X|Y)$$

- WPM-Source: Arbitrarily Varying Sources [Ahl79]

$$R^{WPM}_{X|Y} = \sup_{P(X,Y) \in \text{Conv}(\{P_\pi(X,Y)\}_{\pi \in \mathcal{P}_\pi})} H(X|Y)$$

Practical coding scheme

- M-Source : Standard soft LDPC decoder :

$$P(X,Y) = \sum_{\pi \in \mathcal{P}_\pi} P(\pi) P(X,Y|\pi) \ .$$

- WPM-Source : Standard hard LDPC decoder
  Loss in performance

Experiments

Table: Theoretical (Th.) and practical (Prac.) rate bounds in bit/symbol when X is a BSS and $P(Y|X)$ a BSC.

Source	Conditions	Th. Rate	Prac. Rate
P-Source	$\mathcal{P}_\theta = [0.1, 0.21]$	0.74	0.75
WP-Source	$\mathcal{P}_\theta = [0.1, 0.21]$	0.74	0.75
M-Source	$\mathcal{P}_\pi = \{0.1, 0.21\}$ $p = 0.143$	0.59	0.6
WPM-Source	$\mathcal{P}_\pi = \{0.1, 0.143\}$	0.59	0.75
No SI	$P(X = 1) = 0.5$	1	1

P,WP-Sources : $(0.093x^3 + 0.720x^4 + 0.187x^5, x^5)$
M-Source : $(0.099x^3 + 0.712x^4 + 0.174x^5 + 0.015x^6, x^5)$
WPM-Source : (3,4)

Conclusion and Perspectives

- Practical coding scheme for each model

- Analysis to extend to the non-binary case
- How to avoid the learning sequence?
- How to build a coding scheme for the WPM-Source that performs close to the entropy?

Bibliography

[Ahl79] R. Ahlswede.
Coloring hypergraphs: A new approach to multi-user source coding-1.
Journal of Combinatorics, 4(1):76–115, 1979.

[Csi82] I. Csiszar.
Linear codes for sources and source networks: Error exponents, universal coding.
IEEE Trans. on Inf. Th., 28(4):585–592, 1982.

[Han03] T.S. Han.
Information-spectrum methods in information theory.
Springer, 2003.

[SW73] D. Slepian and J. Wolf.
Noiseless coding of correlated information sources.
IEEE Trans. on Inf. Th., 19(4):471–480, July 1973.